Initial baryon number fluctuations and their fluid dynamical response

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In collaboration with Stefan Floerchinger

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Opportunities for Exploring Longitudinal Dynamics in Heavy Ion Collisions
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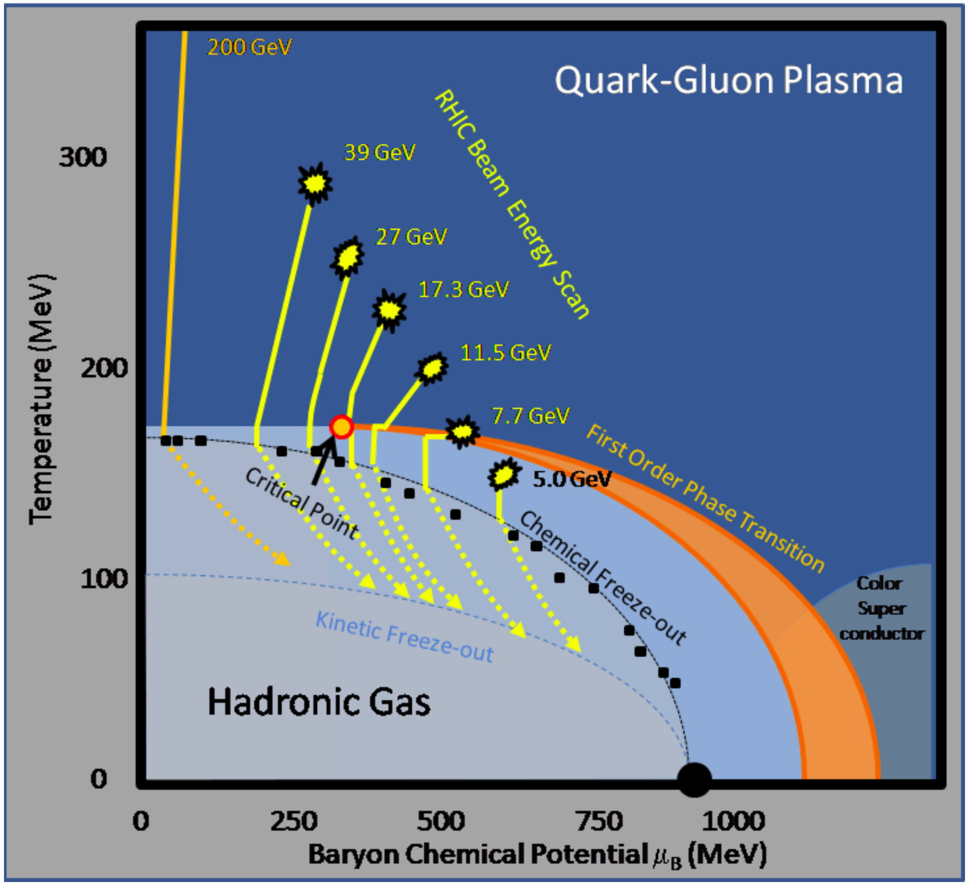
The Ohio State University

Exploring the QCD phase diagram

The location of the critical point of QCD can be determined experimentally by varying the beam energy

- Cumulants of particle multiplicity distributions are sensitive to the critical correlation length.
- Transport coefficients are also sensitive to the critical correlation length
  ⇛ see talks by Yi Yin and J. Noronha



Sources of fluctuations in HIC

- Initial State Fluctuations

In this talk!!!

- Hydrodynamic noise

$\Rrightarrow$ see talks by C. Young and H. Grönqvist

- Fluctuations induced by hard processes
- Freeze-out fluctuations

Background fluctuating splitting

The particle spectrum can be decomposed as

$$E\frac{dN}{d^3p} = E\frac{dN^{back.}}{d^3p} + E\frac{dN^{fluct.}}{d^3p}$$

The background component of the spectrum:

$$E\frac{dN^{back.}}{d^3p} = \frac{1}{(2\pi)^3} p_\mu \int_{\Sigma_f} d\Sigma^\mu \, f(p^\mu; T(x), u^\mu(x), \pi^{\mu\nu}(x), \pi_{\text{bulk}}(x))$$

The fluctuating component of the spectrum

$$E\frac{dN^{fluct.}}{d^3p} = E\frac{dN^{back.}}{d^3p} \left(\sum_{m=-\infty}^{\infty} \sum_{l=1}^{\infty} \tilde{w}_l^{(m)} \, e^{im\phi} \, \tilde{S}_l^{(m)}(p_T) \right)$$

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p_T dependence of each particular mode

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Weights of each mode

$$w_l^{(m)} \sim \langle w_l^{(m)} \rangle_{ini.}$$

p_T dependence of each particular mode

“Simple” question:

- Which of the initial modes $\langle w_l^{(m)} \rangle_{ini.}$ survive the space-time evolution?
- What happens to the space-time evolution of different modes in the presence of a finite chemical potential in an evolving background?

Constitutive equations

The Eqs. of motion are obtained from the conservation laws

$$\mathrm{D}_\mu T^{\mu\nu} = 0 \qquad\qquad D_\mu N^\mu = 0$$

The background-fluctuating splitting indicates that

$$T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu}$$

$$N^\mu = N_0^\mu + \delta N^\mu$$

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For the background hydrodynamical fields we have

$$T_0^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\mathrm{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$N_0^\mu = n\, u^\mu + \nu^\mu$$

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$$\pi^{\mu\nu} = -2\eta\,\sigma^{\mu\nu} = -2\eta\left[\frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta} + \frac{1}{2}\Delta^{\mu\beta}\Delta^{\nu\alpha} - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}\right]\nabla_\alpha u_\beta,$$

$$\pi_{\mathrm{bulk}} = -\zeta\,\theta = -\zeta\,\nabla_\mu u^\mu,$$

$$\nu^\alpha = -\kappa\left[\frac{nT}{\epsilon+p}\right]^2 \iota^\alpha = -\kappa\left[\frac{nT}{\epsilon+p}\right]^2 \Delta^{\alpha\beta}\partial_\beta\left(\frac{\mu}{T}\right).$$

Constitutive equations

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For the fluctuating hydrodynamical fields we have

$$\delta T^{\mu\nu} = \begin{cases} \delta\epsilon \\ \delta u^\mu \\ \delta\pi^{\mu\nu} \\ \delta\pi_{bulk} \end{cases} \qquad\qquad \delta N^\mu = \begin{cases} \delta n \\ \delta u^\mu \\ \delta\nu^\mu \end{cases}$$

Constitutive equations

The Eqs. of motion are obtained from the conservation laws

$$\mathrm{D}_\mu T^{\mu\nu} = 0 \qquad D_\mu N^\mu = 0$$

The background-fluctuating splitting indicates that

$$T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu}$$
$$N^\mu = N_0^\mu + \delta N^\mu$$

One requires to solve equations for the background + fluctuating fields

$$D_\mu T_0^{\mu\nu} = 0$$
$$D_\mu N_0^\mu = 0$$

No back-reaction effects

$$D_\mu \delta T^{\mu\nu} = 0$$
$$D_\mu \delta N_0^\mu = 0$$

Couplings between different fluctuating fields + couplings with background fields

Background fields: Bjorken expansion

- Consider that the EOS $P=P(T,\mu)$

  1. Ideal EOS
  2. Lattice QCD with Taylor expansion

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- The evolution of the background fields is given by the Bjorken expansion
- Parametrize the transport coefficients (AdS/CFT):

$$\frac{\eta(T,\mu)}{s(T,\mu)} = \frac{c}{4\pi} \qquad \zeta = 2\eta(T,\mu)\left(1 - c_s^2(T,\mu)\right) \qquad \kappa = 8\pi^2 \frac{T}{\mu^2}\eta(T,\mu)$$

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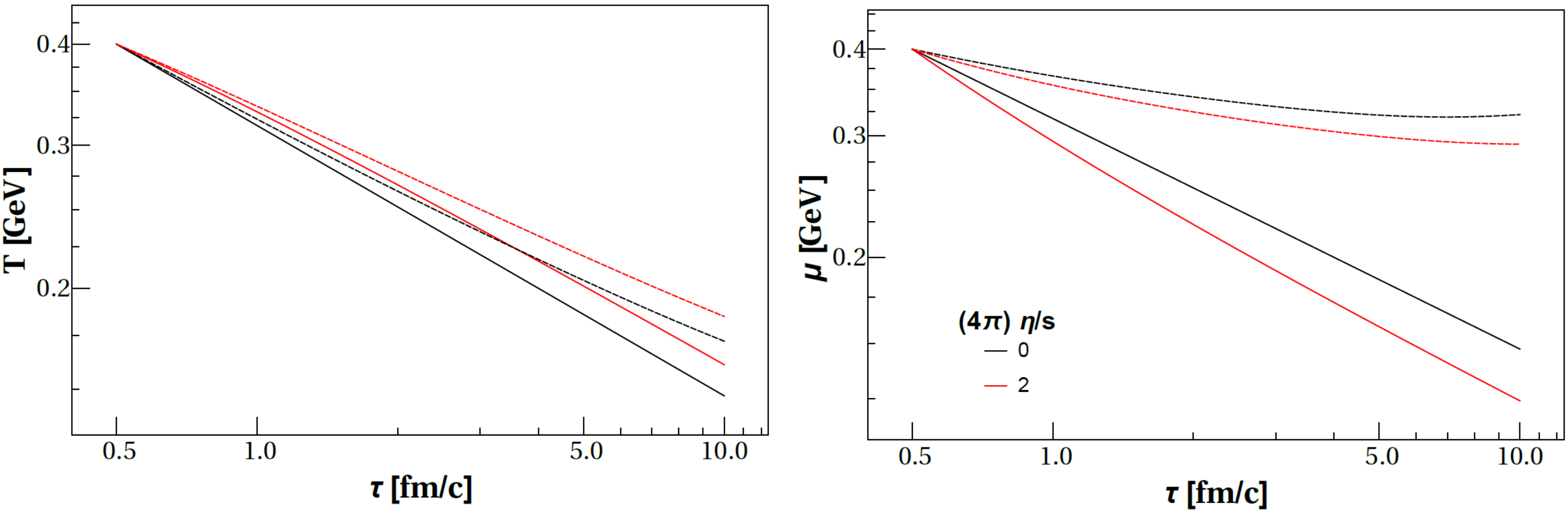

$\tau_0 = 0.5 \text{ fm/c} \qquad \mu_0 = 0.4 \text{ GeV} \qquad T_0 = 0.4 \text{ GeV}$

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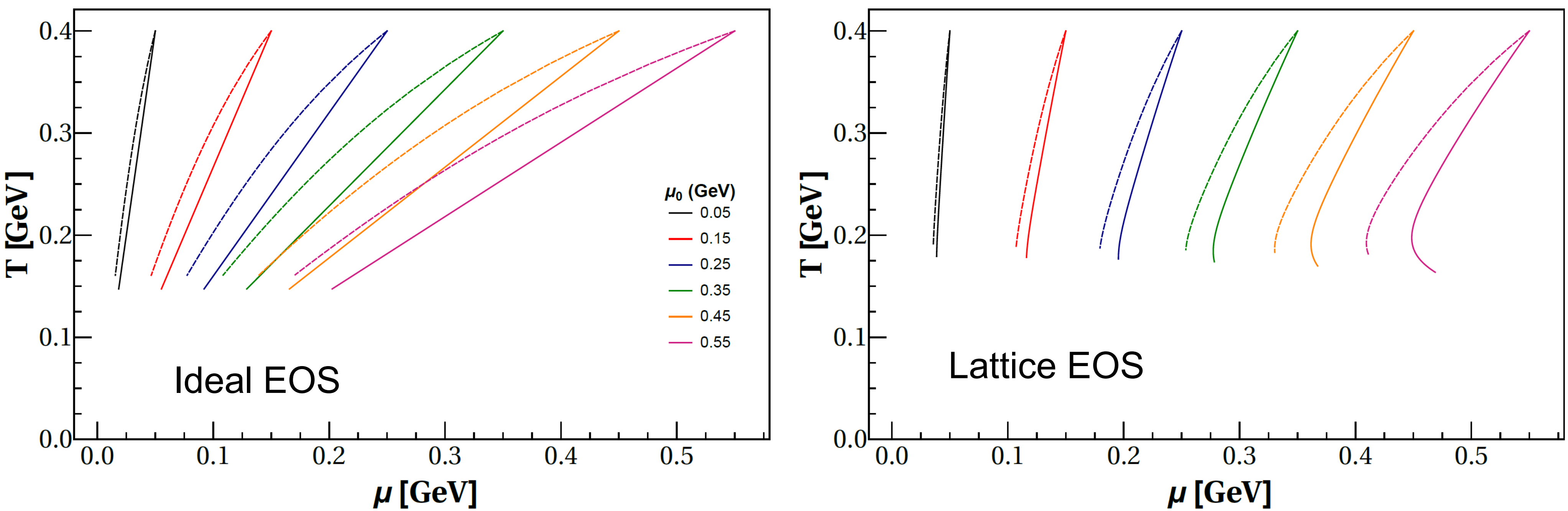

Bessel Fourier decomposition

For solving the equations of the fluctuating fields, it is convenient to use the Bessel-Fourier decomposition, e.g.,

$$\delta\epsilon(\tau, r, \phi, \eta) = \int_0^\infty dk\, k \sum_{m=-\infty}^{\infty} \int \frac{dq}{2\pi}\, \delta\epsilon(\tau, k, m, q)\, e^{i(m\phi + q\eta)}\, J_m(kr),$$

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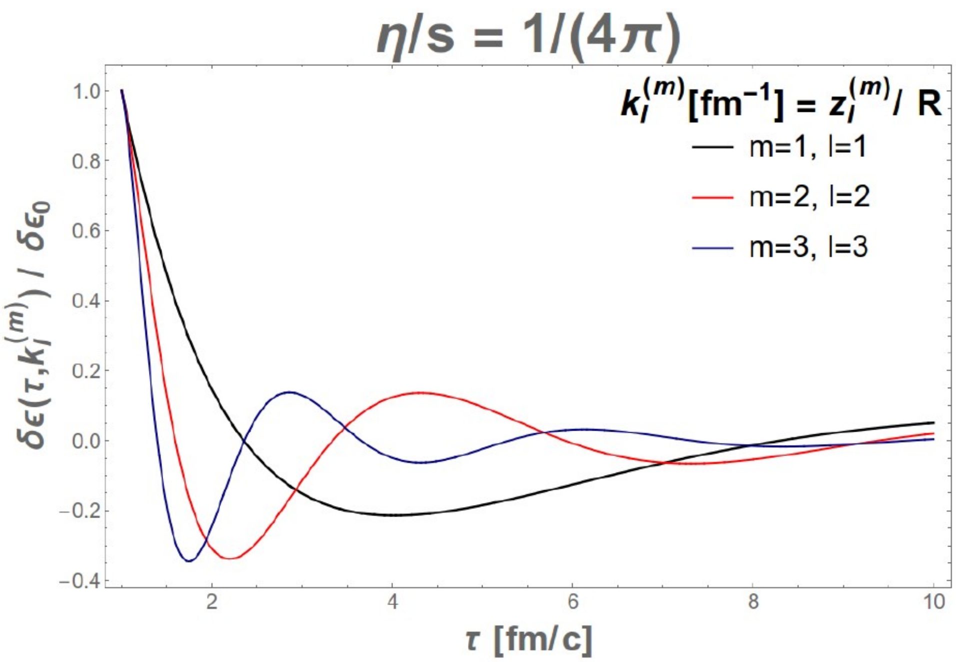
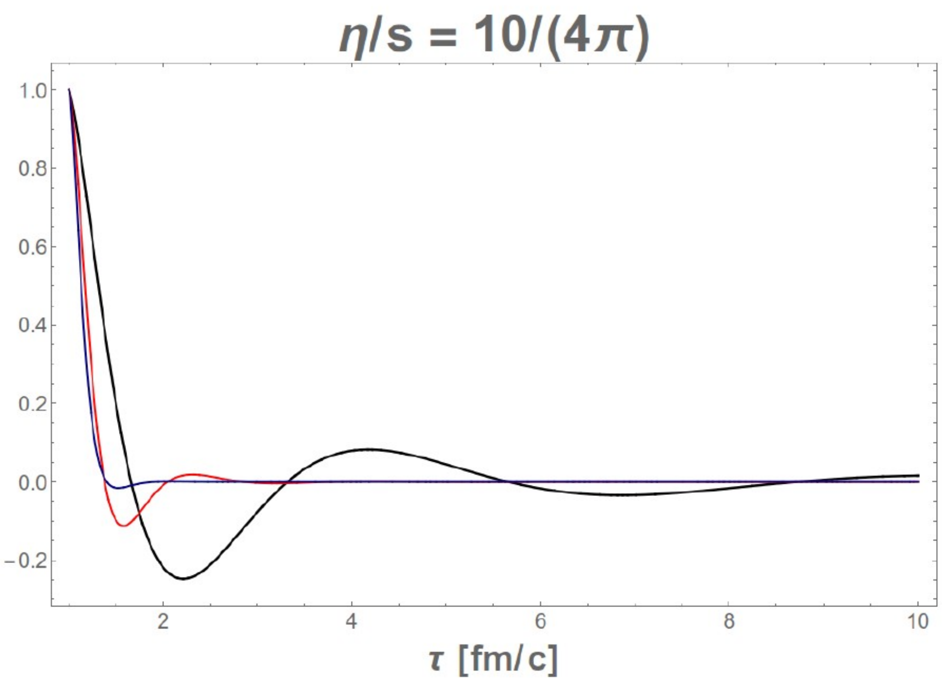
For instance, for the Bjorken flow the evolution equation of $\delta\epsilon(\tau, k, m, q)$ evolves

$$\begin{aligned}
\partial_\tau \delta\epsilon &+ \left[\frac{1}{\tau} + \frac{1}{\tau}\left(\frac{\partial p}{\partial \epsilon}\right)_n - \frac{1}{\tau^2}\left(\frac{\partial \zeta}{\partial \epsilon}\right)_n - \frac{4}{3\tau^2}\left(\frac{\partial \eta}{\partial \epsilon}\right)_n\right]\delta\epsilon \\
&+ \left[\frac{1}{\tau}\left(\frac{\partial p}{\partial n}\right)_\epsilon - \frac{1}{\tau^2}\left(\frac{\partial \zeta}{\partial n}\right)_\epsilon - \frac{4}{3\tau^2}\left(\frac{\partial \eta}{\partial n}\right)_\epsilon\right]\delta n \\
&+ \left[\bar\epsilon + \bar p - \frac{2}{\tau}\bar\zeta + \frac{4}{3\tau}\bar\eta\right]\left(\frac{k}{\sqrt{2}}\left(\delta u^+ - \delta u^-\right) + iq\,\delta u^\eta\right) - \frac{4}{\tau}\bar\eta\, iq\,\delta u^\eta = 0.
\end{aligned}$$

Propagation of transverse modes





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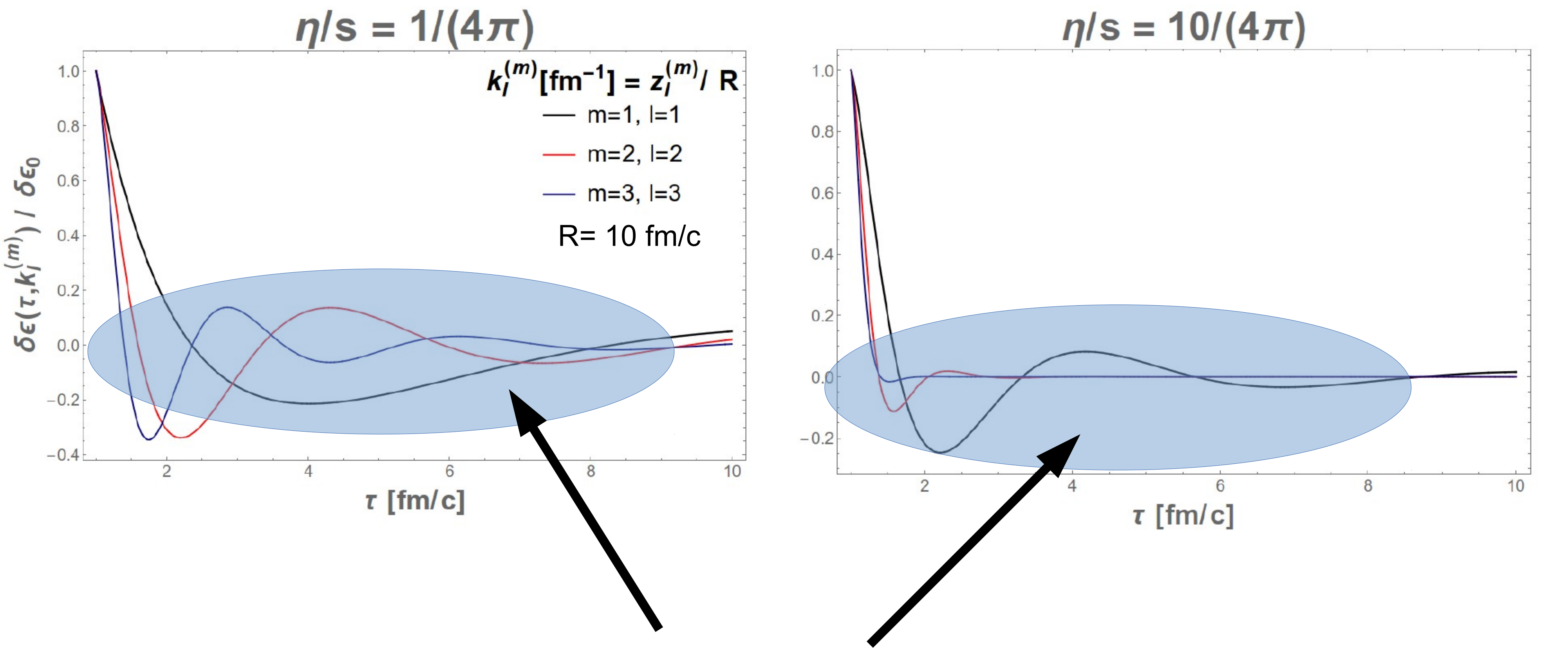
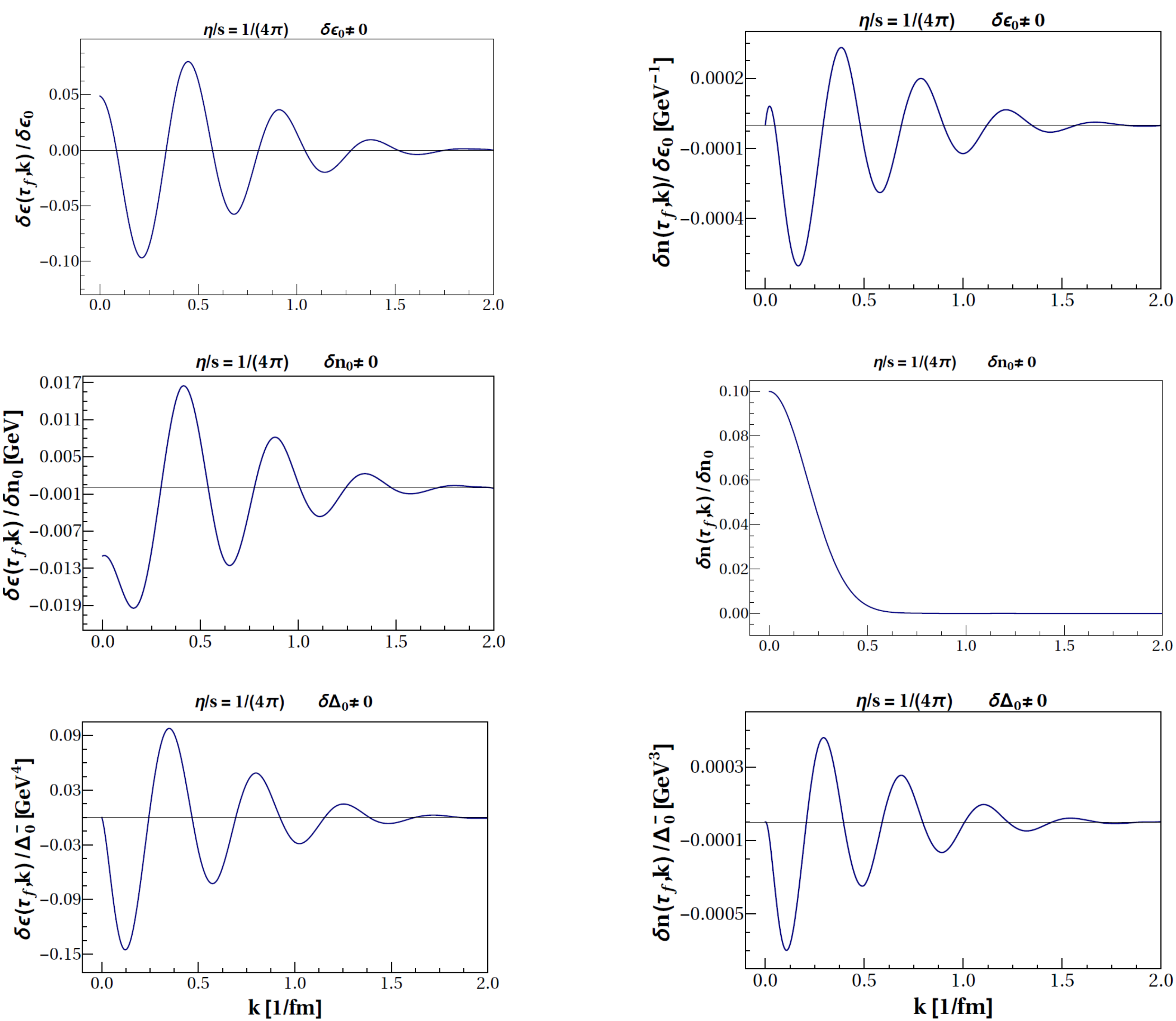


- The oscillation frequency depends on the wave number k
- As one increases η / s the amplitude of the perturbation decrease

Propagation of transverse modes



$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Propagation of transverse modes

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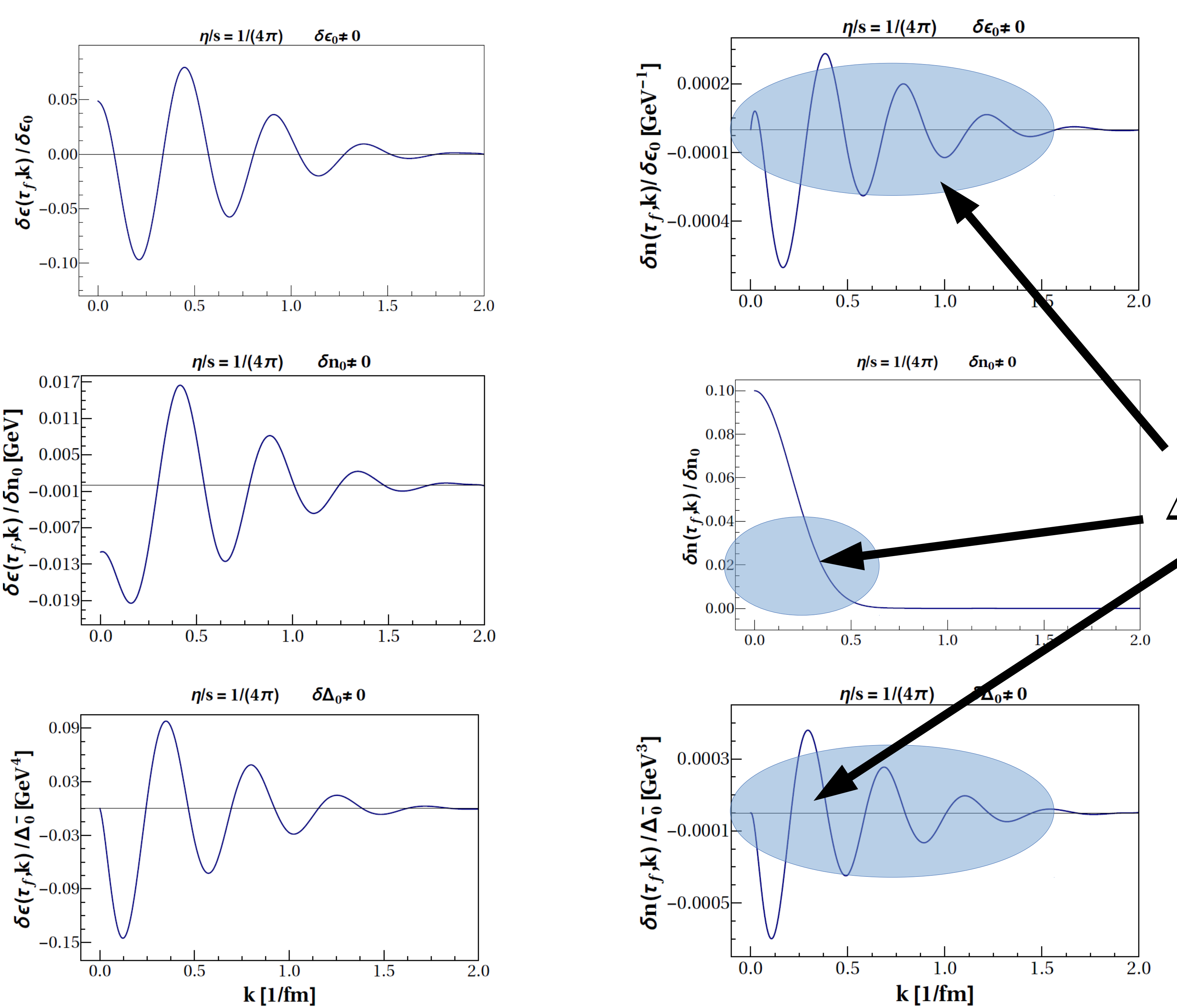
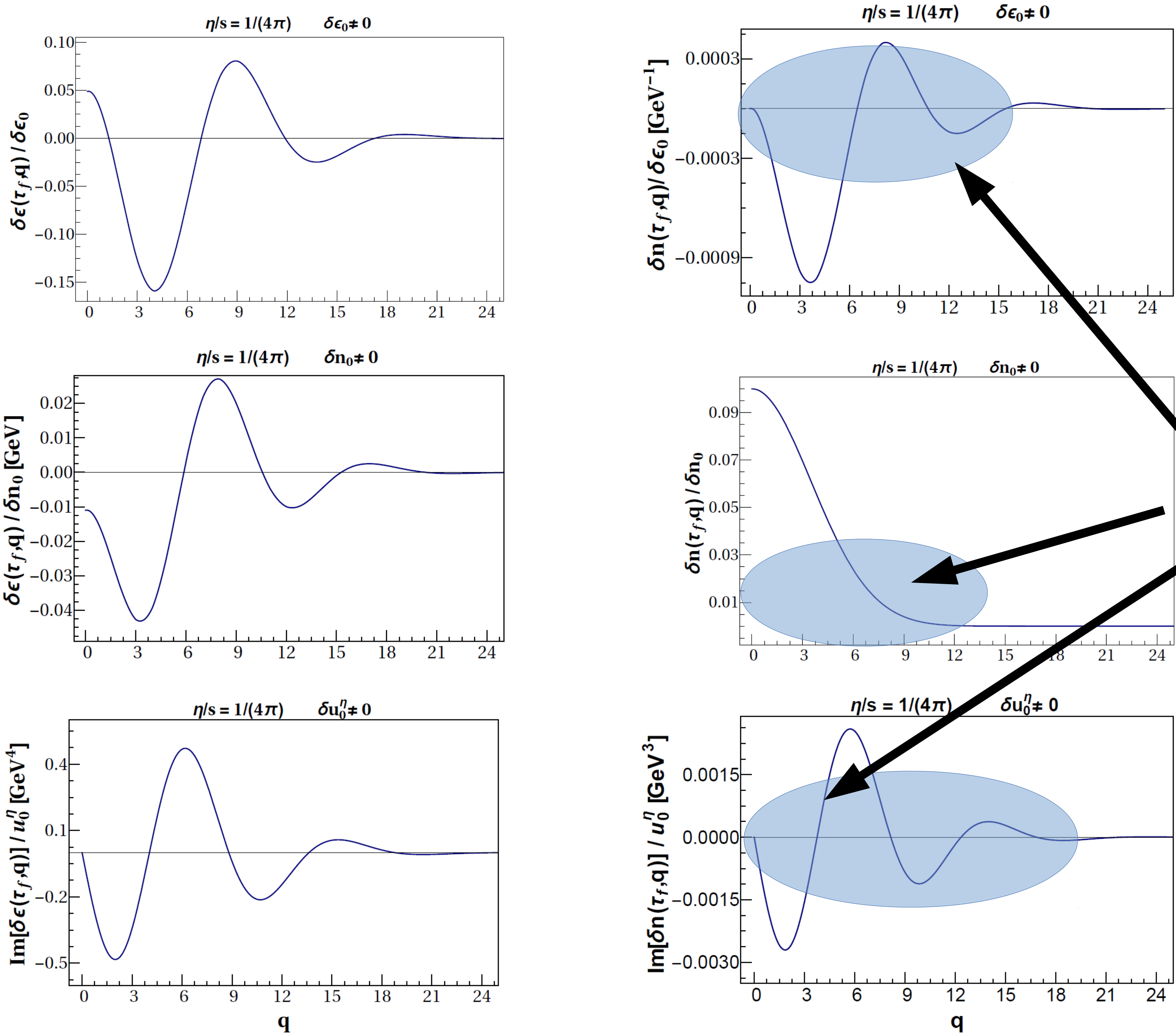


$$\Delta r \sim (\Delta k)^{-1}$$

Important at late times

Propagation of longitudinal modes

$$\frac{\eta}{s} = \frac{1}{4\pi}$$



$$\Delta\eta \sim (\Delta q)^{-1}$$

Important at early times

Baryonic correlation function

In the case of small values of the baryon density

$\Rightarrow$ the equation of motion for the perturbation of the

baryon density decouples

$$\delta n(\tau,k,m,q) = \left(\frac{\tau_0}{\tau}\right)\exp\left[-\frac{m^2}{R^2}I_1(\tau,\tau_0) - q^2 I_2(\tau,\tau_0)\right]\delta n(\tau_0,k,m,q),$$

$$I_1(\tau,\tau_0) = \int_{\tau_0}^{\tau} d\tau' \,\bar{\kappa}\left[\frac{\bar{n}\bar{T}}{\bar{\epsilon}+\bar{p}}\right]^2 \left(\frac{\partial(\mu/T)}{\partial n}\right)_{\epsilon},$$

$$I_2(\tau,\tau_0) = \int_{\tau_0}^{\tau} \frac{d\tau}{\tau'^2} \,\bar{\kappa}\left[\frac{\bar{n}\bar{T}}{\bar{\epsilon}+\bar{p}}\right]^2 \left(\frac{\partial(\mu/T)}{\partial n}\right)_{\epsilon}.$$

Life time of the Expanding plasma

Transport properties

Thermodynamical susceptibility

Baryonic correlation function

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The initial “granularity” of the baryon density

$$\delta n(\tau_0,r,\phi,\eta)=\sum_{m=-\infty}^{\infty}\sum_{l=1}^{\infty}\int\frac{dq}{2\pi}\,\delta n_l^{(m)}(q)\,e^{im\phi+iq\eta}J_m\left(z_l^{(m)}\rho(r)\right)$$

Baryonic correlation function

$$C_{\mathrm{B}}(\phi_1-\phi_2,\eta_1-\eta_2)=\langle n_{\mathrm{B}}(\phi_1,\eta_1)n_{\mathrm{B}}(\phi_2,\eta_2)\rangle_c$$

$$=\sum_{m=-\infty}^{\infty}\int\frac{dq}{2\pi}\,\langle n_{\mathrm{B}}^{(m)}(q)n_{\mathrm{B}}^{(m)}(q)\rangle\,e^{im(\phi_1-\phi_2)+iq(\eta_1-\eta_2)}$$

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At the freeze out the baryon number distribution is proportional to the weights within the linear response

$$n_{\mathrm{Baryons}}^{(m)}(q)=\sum_l S_{\mathrm{Baryons};(m)l}(q)\delta n_l^{(m)}(q)$$

Baryonic correlation function

$$C_{\mathrm{B}}(\phi_1 - \phi_2, \eta_1 - \eta_2) = \langle n_{\mathrm{B}}(\phi_1, \eta_1) n_{\mathrm{B}}(\phi_2, \eta_2) \rangle_c$$

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At the freeze out the baryon number distribution is proportional to the weights within the linear response

$$n_{\mathrm{Baryons}}^{(m)}(q) = \sum_l S_{\mathrm{Baryons};(m)l}(q) \delta n_l^{(m)}(q)$$

$$\Rightarrow \langle n_{\mathrm{B}}^{(m)}(q) n_{\mathrm{B}}^{(m)}(q) \rangle \approx \exp(-2m^2 I_1' - 2q^2 I_2') \langle \delta n_l^{(m)}(q) \delta n_l^{(m)}(q) \rangle$$

$$I_1' \approx \int_{\tau_0}^{\tau_f} d\tau \, \frac{1}{R^2} \, \bar{\kappa} \left[\frac{\bar{n} \bar{T}}{\bar{\epsilon} + \bar{p}} \right]^2 \left(\frac{\partial(\mu/T)}{\partial n} \right)_{\epsilon},$$

→ Relevant at late times

$$I_2' \approx \int_{\tau_0}^{\tau_f} d\tau \, \frac{1}{\tau^2} \, \bar{\kappa} \left[\frac{\bar{n} \bar{T}}{\bar{\epsilon} + \bar{p}} \right]^2 \left(\frac{\partial(\mu/T)}{\partial n} \right)_{\epsilon}.$$

→ Relevant at early times
Baryon ridge?

Conclusions

- We discuss the evolution of the fluid dynamical equations of the background and fluctuating hydrodynamical fields in the presence of a finite chemical potential.
- There are characteristic differences in the dependencies of the perturbations on longitudinal and transverse modes.
- Two particle correlation function of baryonic particles as a function of the difference of azimuthal angles and rapidities provides information of the transport properties and mode propagation in the medium.

Outlook and Perspectives

- Effects of baryon number fluctuations at the freeze-out surface.
- Include second order transport coefficients in the evolution equations of the fluctuating fields
- Study non linear evolution of the perturbations

Backup slides

Hydrodynamics with finite density

For a system with a conserved charge (e.g. baryonic number)

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\rm bulk})\Delta^{\mu\nu} + \pi^{\mu\nu}\,,$$
$$N^\mu = n\, u^\mu + \nu^\mu .$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

From the conservation laws, the equations of motion are (Landau frame)

$$D\epsilon + (\epsilon + p + \pi_{\rm bulk})\nabla_\mu u^\mu + \pi^{\mu\nu}\nabla_\mu u_\nu = 0,$$
$$(\epsilon + p + \pi_{\rm bulk})\, Du^\nu + \Delta^{\nu\mu}\,\partial_\mu(p + \pi_{\rm bulk}) + \Delta^\nu{}_\alpha \nabla_\mu \pi^{\mu\alpha} = 0,$$
$$Dn + n\nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0\,.$$

Equation of state

In the grand canonical ensemble one has P=P(T,μ)

In our work we make use of the ideal EOS one has

$$p(T,\mu) = \frac{1}{4!}a_1\,T^4 + \frac{1}{4}a_2\,T^2\mu^2 + \frac{1}{4!}a_3\,\mu^4$$

$$a_1 = \frac{8\pi^2}{15}\left(N_C^2 - 1 + \frac{7}{4}N_C N_F\right),$$

$$a_2 = \frac{2N_C N_F}{27},$$

$$a_3 = \frac{2N_C N_F}{81\pi^2}.$$

However, we can use more general equation of state from recent lattice data (BNL-Bielefeld collaboration, Wuppertal-Budapest) or analytical results from HTL (Strickland et. al, Vuorinen)

Equation of background hydro. fields

$$\left[T\frac{\partial^2 p}{\partial T^2}+\mu\frac{\partial^2 p}{\partial T\partial\mu}\right]DT+\left[T\frac{\partial^2 p}{\partial T\partial\mu}+\mu\frac{\partial^2 p}{\partial\mu^2}\right]D\mu+(\epsilon+p)\,\theta-2\eta\,\sigma_{\alpha\beta}\sigma^{\alpha\beta}-\zeta\,\theta^2=0,$$

$$(\epsilon+p)\,Du^\nu+\Delta^{\nu\alpha}(s\,\partial_\alpha T+n\,\partial_\alpha\mu)-\Delta^\nu{}_\alpha\nabla_\beta\left(2\,\eta\,\sigma^{\alpha\beta}+\zeta\,\Delta^{\alpha\beta}\,\nabla_\gamma u^\gamma\right)=0\,,$$

$$\frac{\partial^2 p}{\partial T\partial\mu}DT+\frac{\partial^2 p}{\partial\mu^2}D\mu+n\,\theta+\nabla_\alpha\nu^\alpha=0\,.$$

Estimates of the transport coefficients

Transport coefficient	Weakly-coupled QCD	Strongly-coupled theories
η	$k \frac{T^3}{g^4 \log(1/g)}$	$\frac{s(T,\mu)}{4\pi}$
ζ	$15\,\eta(T)\left(\frac{1}{3} - c_s^2(T)\right)^2$	$2\,\eta(T,\mu)\left(\frac{1}{3} - c_s^2(T,\mu)\right)$
κ	$\sim \mu^2/g^4$ for $\mu \gg T$ $\sim T^4/(g^4\mu^2)$for $\mu \ll T$	$8\pi^2 \frac{T}{\mu^2}\eta(T,\mu)$

For practical purposes we use the strongly coupled relations for the transport coefficients

New calculations of the transport coefficients for strongly coupled systems with finite chemical potential.
See Rougemond and Noronha, arXiv:1507.06556